Addendum to 'Soliton solutions for Dirac equations with homogeneous nonlinearity in (1+1) dimensions'

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COMMENT

Addendum to 'Soliton solutions for Dirac equations with homogeneous non-linearity in (1+1) dimensions'

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Abstract. Some results concerning the existence of static soliton solutions are corrected and illustrated by a general example.

In the letter by Mathieu (1985) some side comments, which do not refer to the main results of the work, contain incorrect statements. Firstly, for models satisfying equation (18) of the above reference (hereafter referred to as I), it is not true for all soliton solutions that the energy and the charge become infinite as the frequency approaches zero (bottom of p L1063). This is certainly true for a scalar non-linearity but the simplest counter-example is the vectorial quartic non-linearity. This error originates from an incorrect expansion of the solution as $\omega \to 0$. The assertion also implicitly assumed that the existence of solutions for $\omega \to 0$ can be inferred from the existence of solutions for $\omega \to \mu$. This is not true in general. These remarks invalidate the first part of the second paragraph and affect the last one. I apologise for the confusion concerning this point.

These remarks will be illustrated by an example. Consider the general quartic non-linearity

$$U(\bar{\psi}, \psi) = -\lambda[(\bar{\psi}\psi)^2 + b(\bar{\psi}\mathrm{i}\gamma^5\psi)^2] \tag{1}$$

for which positive frequency soliton solutions exist only if $\omega < \mu$. From equation (21) of I, this implies that $\lambda > 0$. Solutions will now be studied as a function of the parameter b. In particular, the dependence upon b of the allowed frequency range for soliton solutions will be determined, which will then provide the class of quartic non-linearities of the form (1) admitting static solutions. The solution is given by equations (14), (17) and (19) of I:

$$\psi(x, t) = \left(\frac{\omega - \mu}{U(\bar{\chi}_0, \chi_0)}\right)^{1/2} \chi_0 \, \mathrm{e}^{-\mathrm{i}\omega t}$$

$$\chi_0(x) = \begin{pmatrix} \cosh \alpha x \\ \beta \sinh \alpha x \end{pmatrix}$$

$$\alpha = (\mu^2 - \omega^2)^{1/2} \qquad \beta = [(\mu - \omega)/(\mu + \omega)]^{1/2}.$$

A necessary condition for the existence of solutions is $U(\bar{\chi}_0, \chi_0) < 0$ or

$$[(1-\beta^2)^2 + 4b\beta^2] \cosh^4 \alpha x + [(1-\beta^2) - 2b]2\beta^2 \cosh^2 \alpha x + \beta^4 > 0.$$

Clearly the condition is always satisfied for $b \geqslant 0$. For $b < 0$, the above inequality can be satisfied everywhere only if

$$b \geqslant -(1-\beta^2)^2/4\beta^2. \tag{2}$$

For a static solution, $\omega = 0$ and $\beta = 1$, and this requires $b \geqslant 0$. However, when $b = 0$, $U(\bar{\chi}_0, \chi_0)$ is a constant and the solution is not localised. One verifies that the corresponding charge is infinite. (Hence the conditions $\omega < \mu$ and $U(\bar{\chi}_0, \chi_0)$ strictly negative are not sufficient to ensure the existence of static localised solutions.) On the other hand, for $b > 0$, the solution exists and the charge is finite. The case $b = 1$ corresponds to the vectorial non-linearity.

For $b < 0$, there are no static solutions. Furthermore solutions do not exist for ω arbitrarily close to zero. Indeed the inequality (2) restricts the possible frequency range to

$$0 < \beta^2 < 1 + 2|b| - 2(b^2 + |b|)^{1/2}.$$

I would like to thank J Stubbe for sending me a copy of his work (Stubbe 1985), which led me to reconsider the statements related to the existence of static solutions.

References

Mathieu P 1985 *J. Phys. A: Math. Gen.* **18** L1061
Stubbe J 1985 *Bielefeld University preprint* BI-TP 85/27